Computational Electromagnetics and Fast Physical Optics

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Abstract

A new hybrid technique is used to compute the RCS pattern of a spherical cap by means of PO approximation. The approach used here combines analytical techniques with path deformation techniques to compute accurately the full RCS diagram. The computation time is independent with frequency. This approach is faster than standard numerical techniques (PO integration) and more accurate than non-uniform asymptotic techniques.

Keywords: Physical Optics, Radar Cross Section, Highly oscillatory integrals, path deformation.

1. Introduction

Computational science and computational engineering have been developed from the very invention of the modern computer. Since the application of the Neumann's machine to the solution of termonuclear and nuclear physics problems, the use of computers was extended to almost all engineering processes.

The Field of Electromagnetic Wave Theory produces a great number of challenging problems for Electrical and Electronic Engineers. Among them, scattering problems are particularly hard. New technologies on wireless communications, remote sensing, space communication systems, earth observation satellites… require precise designs that usually overwhelm the accuracy of the existing computational techniques. The international community on Antennas and Propagation has been developing for many years a lot of very interesting algorithms to solve fast and efficiently a large amount of problems arising in telecommunications: Antenna reflectors, RCS of planes … Nevertheless, in many cases the accuracy and the computation time are far form satisfactory.

In mathematics and physics, the scattering theory is a framework for studying and understanding the scattering of waves and particles. Prosaically, wave scattering corresponds to the collision and scattering of waves with some material object. More precisely, scattering consist of the study of how solutions of partial differential equations, propagating freely "in the distant past", come together and interact with one another or with a boundary condition, and then propagate away "to the distant future". In acoustics, the differential equation is the wave equation, and scattering studies how its solutions, the sound waves, scatter from solid objects or propagate through non-uniform media (such as sound waves, in sea water, coming from a submarine). In our case (classical electrodynamics), the differential equations are Maxwell equations, and the scattering of light or radio waves is studied. In quantum and particle physics, the equations are those of quantum electrodynamics QED, quantum chromodynamics QCD and the Standard Model, the solution of which correspond to fundamental particles. In quantum chemistry, the solutions correspond to atoms and molecules, governed by the Schrodinger equation.

Computational techniques for scattering problems in classical electrodynamics can be divided into two different groups:

On one hand High Frequency Techniques. Notable among these are the geometrical theory of diffraction (GTD) introduced by J. B. Keller; the Physical Theory of Diffraction (PTD) developed by P. Y. Ufimtsev; the Uniform Asymptotic Theory (UAT) and the Uniform Theory of Diffraction (UTD) formulated by Lewis et al. and Kouyoumjian and Pathak; and the Spectral Tory of Doffractopm (STD) introduced by Mittra and Others.

On the other hand Numerical Techniques, which are full wave methods. Notable among these are the Method of Moments (MoM), Finite Element Method (FEM), Finite Difference Time Domain (FDTD). The first one is much more suitable for radiation problems while the second one is suitable for guided problems. The third is suitable for wideband problems.

High Frequency Techniques	Numerical Techniques
Ray Methods: Geométrical Optics (GO) Geometrical Theory of Diffraction (GTD) Uniform Asymptotic Theories (UAT,UTD)	**Differential Equation Methods:** Finite-Difference Methods (FDTD) Finite-Element Methods (FEM) Stability and Accuracy
Current Base Methods: Physical Optics (PO) Physical Theory of Diffraction (PTD)	**Integral Equation Methods:** Spatial Domain Spectral Domain Periodic MoM Fast Methods (FMM, AIM)
Incremental Methods: Boundary Waves Methods Incremental Theory of Diffraction (ITD)	**Modal Expansion Methods:** Mode Matching Generalized Admitance Matrix
Complex Rays and Gaussian Beams:	

Table 1. *Computational Techniques*

The following table contains a summary of the main techniques:

Methods from the first column are based on the expansion of the solution in terms of asymptotic series combined with the localization principle. For these methods, the accuracy increases with frequency. As far as the computation time, the first group (Ray Methods) is the fastest one with a CPU time not frequency dependent. The second group (current base methods) is not so fast but more robust since the solution is given by a highly oscillatory integral.

Methods from the second column are based on the discretization of the Maxwell's equations (the integral or the differential form) and the approximation of the solution by a finite dimensional subspace. The accuracy is much higher for these methods than for the high frequency methods. The weakpoint of these methods is the CPU time and the requirement in memory, especially in high frequencies.

Numerical methods are suitable for problems with a reduced electrical size as the computational complexity increase with frequency. On the other hand, High frequency methods are suitable for geometries with a detail which is much larger than the wavelength. Therefore we find that high frequency and numerical techniques are complementary, in the sense that they are suitable for different problems.

Despites the great amount of problems that can be handled by these techniques (numerical and high frequency), there are still some difficult situations where no method can give a completely satisfactory solution.

Hybrid techniques try to combine numerical and high frequency methods to both increase the range of applicability of the numerical methods and increase the accuracy of high frequency methods. Nowadays, hybrid techniques are a hot spot in computational electromagnetics.

1. Physical Optics

The Physical Optics (PO) method is a high frequency approximation that allows obtaining a solution with a high accuracy and efficiency. The Physical optics method consist of approximate the induced current on the scatterer (the solution of the integral equation) by the, so called, PO current, given by the following expression:

$$\vec{J}_s = \begin{cases} 2\cdot\hat{n}_s \times \vec{H}^i & lit \\ 0 & shadow \end{cases}$$

Now, the diffracted field can be obtained explicitly by the integral of the PO current:

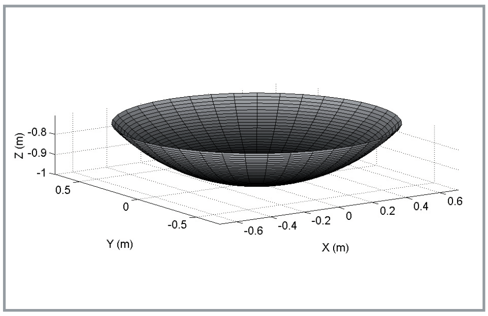
$$\vec{E}_s(\vec{r}) = \frac{j\cdot k\cdot Z_0}{4\pi}\iint_S \hat{k}\times\hat{k}\times\left[2\cdot\hat{n}_s\times\vec{H}^i\right]\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}dS$$

The approximation of the induced current can be justified in different ways. The most typical way is using a reasoning based on fields.

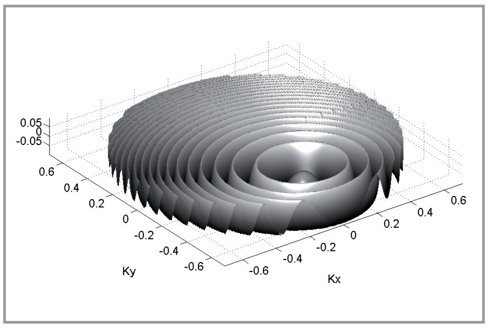
The next step to obtain an estimation of the scattered field $\vec{E}_s$ is to evaluate numerically the integral. For very large frequency, the integral obtained is a highly oscillatory integral. Due to the complexity of the integrand, the computation time using a standard numerical quadrature rule is very large.

In some cases, the PO integral can be computed directly using brute force algorithm. In that case, the method is known as PO method. Nevertheless, in order to speed up the calculation of these integrals, a number of special methods have been developed.

Next we show an example of fast method for computing the PO integral in the case of a semi spherical cap surface:



■ **Figure 1.** *Spherical surface*



■ **Figure 2.** *Highly oscillatory integrand (real part)*

2. Approach

Consider a PEC spherical cap as depicted in fig 1. The parameterization is given in spherical coordinates by the following formulas:

$$\begin{cases} x = \cos\varphi \sin\theta \\ y = \sin\varphi \sin\theta \quad \varphi \in [0, 2\pi], \theta \in [0, \theta_{max}] \\ z = -\cos\theta \end{cases} \tag{1}$$

The incident field is given by the following formula:

$$\vec{H}_i(\vec{r}) = \vec{H}_i^0 e^{-j\vec{k}_i \cdot \vec{r}}$$
$$\vec{E}_i(\vec{r}) = \vec{E}_i^0 e^{-j\vec{k}_i \cdot \vec{r}} \tag{2}$$

We define the scattering amplitude pattern U for the polarization of interest via the copolarized component of the backscattered far field as:

$$U = r \exp(jkr) \frac{H^s \cdot H_0^i}{\left|H_0^i\right|^2} \quad r \to \infty \tag{3}$$

Using the PO approximation, the amplitude pattern can be computed by the following integral expression:

$$U(k,\varphi) = \frac{jk}{2\pi} \iint_S \left(\hat{n}_\alpha \cdot \hat{k}_s\right) e^{j2\vec{k}_s \cdot \vec{r}_\alpha} ds \tag{4}$$

The vector k_s is given by:

$$\hat{k}_s = \sin\phi \cdot \hat{y} + \cos\phi \cdot \hat{z} \tag{5}$$

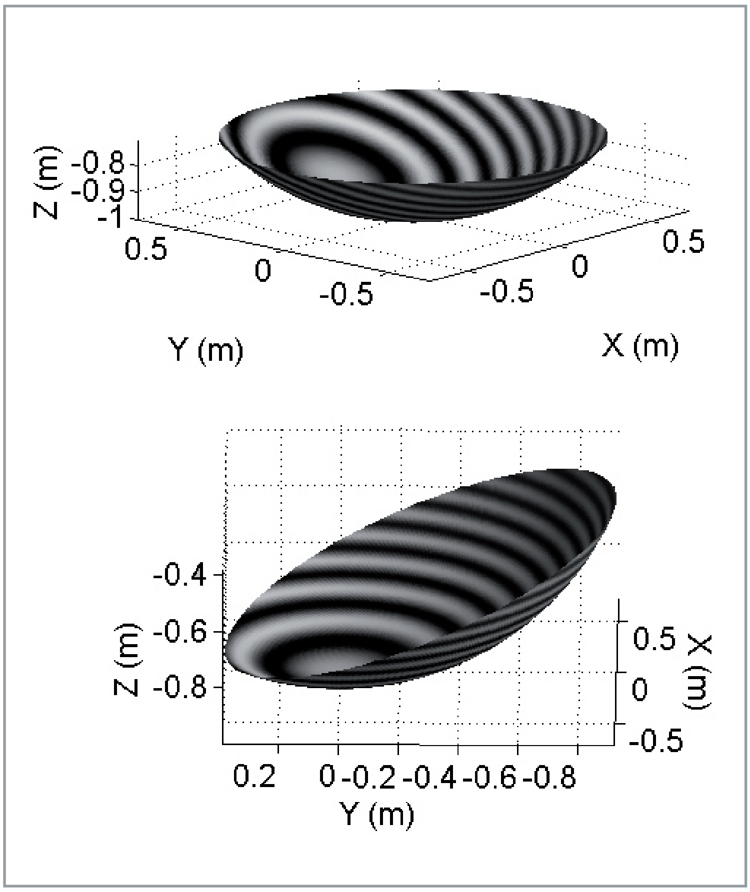
Using spherical coordinates, the integral (4) is given by:

$$U(k,\varphi) = \int_0^{2\pi} \int_0^{\theta_{max}} f(\varphi,\theta,\phi) \sin\theta \exp(2jk \cdot f(\varphi,\theta,\phi)) d\theta d\varphi$$
$$f(\varphi,\theta,\phi) = \sin\varphi \sin\theta \sin\phi + \cos\theta \cos\phi \tag{6}$$

Due to the highly oscillatory nature of the integrand (see fig. 2), the direct computation of the integral using standard quadrature rules is very inefficient.

In order to compute efficiently the integral (6), a combined technique is used. First, the two dimensional integral is reduced to a one dimensional integral as follows:
A rotation in the coordinate system is performed around the x axis to change the ks vector into the z unitary vector (see fig. 3). The integral (6) takes the following analytical expression:

$$U(k,\varphi) = \int_0^{2\pi} \int_0^{\theta'_{max}(\varphi')} \cos\theta' \sin\theta' \exp(2jk \cdot \cos\theta') d\theta' d\varphi' \tag{7}$$



■ **Figure 3.** *Rotation of the coordinate system.*

where the function θ'_{max} is given by:

$$\theta'_{max}(\varphi') = \arccos(\sin\varphi(\varphi')\sin\theta_{max}\sin\phi + \cos\theta_{max}\cos\phi) \quad (8)$$

The relation between φ and φ' is given by:

$$\tan\varphi' = \frac{\cos\phi\sin\varphi\sin\theta_{max} - \sin\phi\cos\theta_{max}}{\cos\varphi\sin\theta_{max}} \quad (9)$$

The integral with θ' can be computed analytically:

$$U(k,\varphi) = \int_{\varphi'_{min}}^{\varphi'_{max}} \left(\int_0^{\theta'_{max}(\varphi')} \cos\theta'\sin\theta'\exp(2jk\cdot\cos\theta')d\theta' \right) d\varphi' =$$
$$= \frac{1}{4k}\int_{\varphi'_{min}}^{\varphi'_{max}} (2jk\cos\theta'-1)\exp(2jk\cdot\cos\theta')\Big|_0^{\theta'_{max}(\varphi')} d\varphi' =$$
$$= \frac{1}{4k}\int_{\varphi'_{min}}^{\varphi'_{max}} (2jk\cos\theta'_{max}(\varphi')-1)\exp(2jk\cdot\cos\theta'_{max}(\varphi'))d\varphi' -$$
$$- \frac{1}{4k}\int_{\varphi'_{min}}^{\varphi'_{max}} (2jk-1)\exp(2jk)d\varphi' =$$
$$= \frac{1}{4k}\int_{\varphi'_{min}}^{\varphi'_{max}} (2jk\cos\theta'_{max}(\varphi')-1)\exp(2jk\cdot\cos\theta'_{max}(\varphi')) -$$
$$- \frac{(2jk-1)}{4k}\exp(2jk)d\varphi' \quad (10)$$

The surface highly oscillatory integral is changed into a line integral (see fig 4) again highly oscillatory. Therefore a computation time improvement has been achieved.

The line integral is computed efficiently using path deformation techniques.

In order to find analytically the path of integration a change of variable is $\varphi'(\varphi)$ performed:

$$\varphi' = \arctan\frac{\cos\phi\sin\varphi\sin\theta_{max} - \sin\phi\cos\theta_{max}}{\cos\varphi\sin\theta_{max}} \quad (11)$$

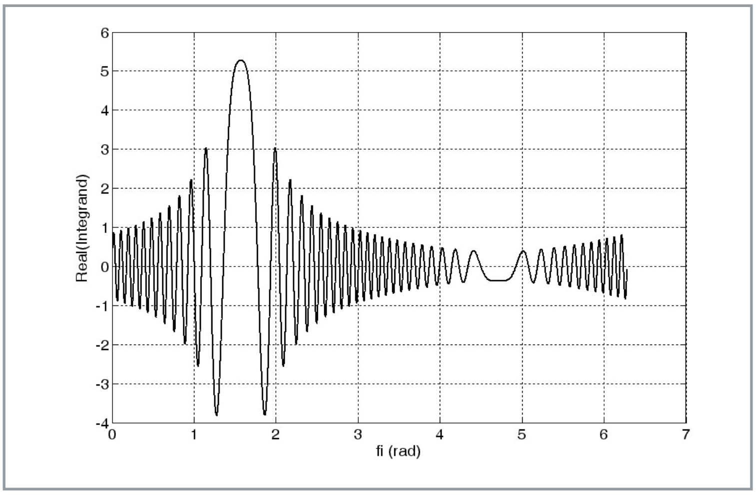


Figure 4. *Highly oscillatory line integral.*

The resulting integral is:

$$\int_0^{2\pi} (2jk\cos\theta'_{max}(\varphi)-1)(d\varphi'/d\varphi)\exp(2jk\cdot\cos\theta'_{max}(\varphi))d\varphi \quad (12)$$

where:

$$\cos\theta'_{max}(\varphi) = \sin\phi\sin\varphi\sin\theta_{max} - \cos\phi\cos\theta_{max}$$
$$(d\varphi'/d\varphi) = (y'\cdot x - x'\cdot y)/(x^2+y^2)$$
$$x = \cos\varphi\sin\theta_{max}$$
$$y = \cos\phi\sin\varphi\sin\theta_{max} - \sin\phi\cos\theta_{max}$$
$$x' = -\sin\varphi\sin\theta_{max}$$
$$y' = \cos\phi\cos\varphi\sin\theta_{max} \quad (13)$$

the integral (12) can be written in a more succinct way:

$$\int_0^{2\pi} f(\varphi+\pi/2)\cdot\exp(jk\cdot(a\cos(\varphi)+b))d\varphi \quad (14)$$

where:

$$f(\varphi) = (2jk\cos\theta'_{max}(\varphi)-1)(d\varphi'/d\varphi) \quad (15)$$

$$a = \sin\phi\sin\theta_{max}$$
$$b = -\cos\phi\cos\theta_{max} \quad (16)$$

A translation of $\pi/2$ was performed in order to locate the stationary phase points at $fi=0$, $fi=\pi$.

The integral (14) is still highly oscillatory, yet, the phase function has a very simple analytical expression. In order to change the integral (14) into a slow varying integral, the path deformation technique is applied. The new path of integration can be found analytically (see fig. 5):

$$C_1(p) = \arccos(1 + jp^2/a)\cdot sign(p)$$
$$C_2(p) = \pi + conj(\arccos(1 + jp^2/a)\cdot sign(p)) \quad (17)$$

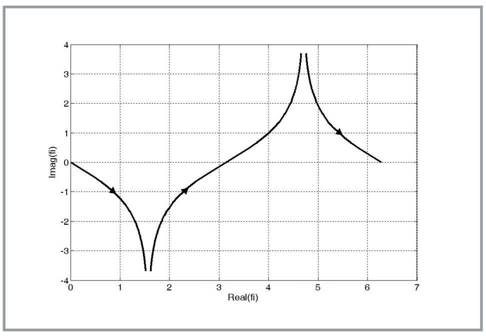


Figure 5. *Path deformation*

The integral (14) can be obtained as a sum of :

$$\int_0^{2\pi} f(\varphi+\pi/2)\cdot\exp(jk\cdot(a\cos(\varphi)+b))d\varphi = I_1 + I_2 \quad (18)$$

where:

$$I_1 = \int_{C1} f(z+\pi/2)\cdot\exp(jk\cdot(a\cos(z)+b))dz$$

$$I_2 = \int_{C2} f(z+\pi/2)\cdot\exp(jk\cdot(a\cos(z)+b))dz \quad (19)$$

Using the parameterization (17) of the paths C_1 and C_2:

$$I_1 = K_1\int_{-\infty}^{+\infty} f(C_1(p)+\pi/2)\cdot(dC_1/dp)\exp(-kp^2)dz$$

$$I_2 = K_2\int_{-\infty}^{+\infty} f(C_2(p)+\pi/2)\cdot(dC_2/dp)\exp(-kp^2)dz$$

$$K_1 = \exp(jk(a+b))$$

$$K_2 = \exp(jk(-a+b)) \quad (20)$$

where:

$$K_1 = \exp(jk(a+b))$$

$$K_2 = \exp(jk(-a+b))$$

$$dC_1/dp = 2j\Big/\sqrt{p^2-2ja}$$

$$dC_2/dp = conj\left(2j\Big/\sqrt{p^2-2ja}\right) \quad (21)$$

See ref [9] for more details about the path deformation.

In the end, the surface highly oscillatory integral (fig. 2) has been changed into slow varying line integrals (fig. 6) without compromising the accuracy. These slow varying integrals can be easily computed by means of standard Gaussian quadrature. Note that the complexity of the integrand in (20) do not increase with frequency, therefore, the same number of sample points

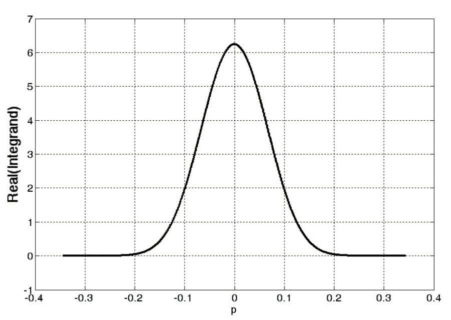


Figure 6. *Slow varying function*

Surface Integral (Highly Oscillatory, 2D)
$U(k,\varphi) = \int_0^{2\pi}\int_0^{\theta_{max}} f(\varphi,\theta,\phi)\sin\theta\exp(2jk\cdot f(\varphi,\theta,\phi))d\theta d\varphi$ $f(\varphi,\theta,\phi) = \sin\varphi\sin\theta\sin\phi + \cos\theta\cos\phi$
Contour Integral (Highly Oscillatory, 1D)
$\int_0^{2\pi} f(\varphi+\pi/2)\cdot\exp(jk\cdot(a\cos(\varphi)+b))d\varphi$
Steepest Descent Integral (Slow varying, 1D)
$I_1 = K_1\int_{-\infty}^{+\infty} f(C_1(p)+\pi/2)\cdot(dC_1/dp)\exp(-kp^2)dz$ $I_2 = K_2\int_{-\infty}^{+\infty} f(C_2(p)+\pi/2)\cdot(dC_2/dp)\exp(-kp^2)dz$

Table 2. *Summary of the Process*

can be used to compute total RCS for different frequencies.

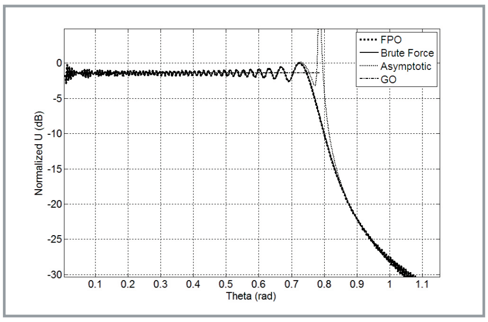
In order to compute the full radiation pattern, the number of sample points must be proportional to the scattering size O(N), a special interpolation technique can be used (see ref [9]) to reduce the computation time of the total radiation pattern from O(N) to O(1).

To summarize, the technique explained in this paper can be split in two parts. First, reduce the 2D highly oscillatory integral into a contour integral (still oscillatory, but one-dimensional). Second, reduce the 1D highly oscillatory integral into a slow varying integral. The following table summarizes the process

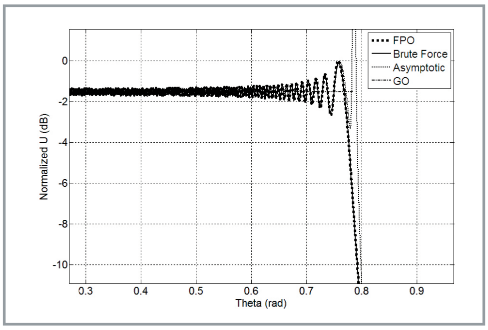
3. Numerical Experiments

Figure 7 shows the RCS pattern for a spherical cap with $\theta max=\pi/4$ and $D=10\lambda$. The figure shows the comparison of the Fast PO method suggested above, a brute force numerical technique and an asymptotic technique (see appendix I for more details). As we can see, a perfect agreement between the numerical technique and the fast method is obtained. The asymptotic method fails to be accurate for those angles where the stationary phase point is close to edge.

Figure 8 and 9 shows the same results for $D=100\lambda$ and $D=500\lambda$. Figure 10 shows the computation time vs the electrical size of the scatter for the Fast PO method and the brute force method. As we can see, the computation time is $O(1)$ for the fast method, and $O(N^3)$ for the brute force

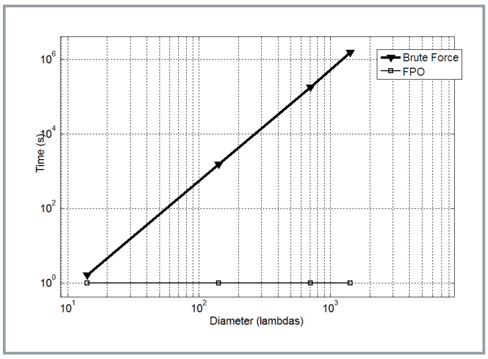


■ **Figure 8.** *RCS pattern for a spherical cap with $\theta max=\pi/4$ and $D=100\lambda$*



■ **Figure 9.** *RCS pattern for a spherical cap with $\theta max=\pi/4$ and $D=1000\lambda$*

numerical method. The computation time for the



■ **Figure 10.** *RCS Computation time (brute force vs fast method)*

brute force method is that because for each sample point, the double integral takes $O(N^2)$ operations, while the amount of sample points is O(N).

5. Conclusions

A method for the efficient calculation of the whole radar cross-section pattern of spherical cap surfaces is presented. It has been shown that the computation time is independent of frequency. Accuracy is also good for all frequencies and incident angles.

Appendix

The Asymptotic technique used to compare the method presented in this paper is the following:

$$\iint f(\theta,\varphi)\exp(jKg(\theta,\varphi))d\theta\, d\varphi : I_s + I_{b1} + I_{b2} \qquad (22)$$

Where Is gives the contribution of the stationary phase point, Ib gives the contribution of the critical boundary points:

$$I_s = \frac{2\pi f^S}{k}\exp(jkg^S)\sqrt{1/\left|g^S_{\theta\theta}g^S_{\varphi\varphi} - g^{S\,2}_{\theta\varphi}\right|}\exp(\frac{\pi}{4}\sigma(\delta+1))ds^S$$
$$I_b = j\frac{f^B}{g^B_\theta k}\exp(jkg^B)\sqrt{\frac{2j\pi}{kg^B_{\varphi\varphi}}}ds^B \qquad (23)$$

See [5] for more details.

6. Acknowledgement

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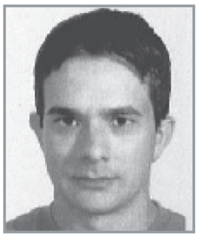

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